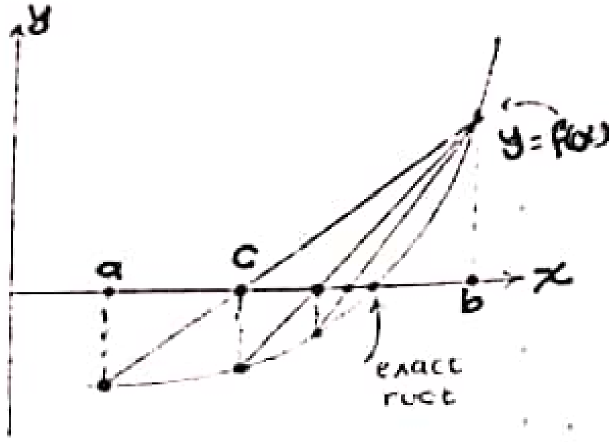
Method of false position طريقة الموقع الخاطئ

لاحظنا ان طريقة التنصيف طريقة سهلة وتمتاز او تمتاز ببساطة تحليلها للخطأ الناتج ولكنها ليست كفوءة تماماً. في العديد من المعادلات يمكن تحسين عملية الوصول الى الحل وتسريعها ـ احدى الطرق المتبعة هي طريقة الموقع الخاطئ او طريقة التقريب بالخط المستقيم والتي تتلخص كالاتي:

Consider the equation $y = f(x) = 0$ and let a, b be two values of x such that $f(a)$ and $f(b)$ are of opposite signs



Let $a<b$. The graph of $y=f(x)$ will meet the x-axis at some point between a and b.

The equation of the straight line joining the two points $[a, f(a)]$ and $[b, f(b)]$ is

$$\frac{y-f(a)}{x-a} = \frac{f(b)-f(a)}{b-a}$$

This straight line intersects with the x axis $(y=0)$ at :

$$x = \frac{a\,f(b) - b\,f(a)}{f(b)-f(a)} = C \quad \text{----} \; *$$

Now:

if $f(a)$ and $f(c)$ are of opposite signs. then the root lies between a and c. otherwise it lies between c and b.

if the root lies between a and c then

$$\text{make } b = c$$
$$f(b) = f(c)$$

and repeat equation (*) agauh

if the root lies between b and c then:

$$\text{make } a = c$$
$$f(a) = c$$

and repeat equation (*) again.

The above method is applied repeatedly till the desired accuracy is obtained that is

$$|C_{n+1} - C_n| \leqslant e$$

Example :

find an approximate value of the root of the equation $x^3+x-1=0$ which lies in the interval $[0.5, 1]$ make two iterations only :

$f(x) = x^3 + x - 1$

$f(0.5) = -0.375$

$f(1) = 1$ opposite signs

$f(0.5) * f(1) = -0.375$ -ve

$\therefore$ The root lies between 0.5 and 1

Now : $a = 0.5$ $b = 1$

$f(0.64) = -0.0979$

$$c = \frac{a f(b) - b f(a)}{f(b) - f(a)} = 0.64$$

Now $f(1) * f(0.64) = -0.0979 < 0$

$\therefore$ The root lies between 0.64 and 1

$a = c$ $f(a) = c$

Now : $a = 0.64$ $b = 1$

$f(a) = c$

$$c = \frac{(0.64)(1) - (1)(-0.0979)}{1 - (-0.0979)} = 0.672$$

$\therefore$ The approximate root is 0.672

Example :

Find the real root of equation in the range [2, 3]

$x \cdot \log_{10} x - 1.2 = 0$

use 3 iterations

$f(x) = x \log_{10} x - 1.2$

$f(2) = -0.6$

$f(3) = 0.23$

The root lies between 2 and 3 and it is near to 3

$a = 2 \qquad b = 3$

$$c = \frac{a f(b) - b f(a)}{f(b) - f(a)} = 2.723$$

$f(c) = -0.04 \qquad c = 2.72 \qquad b = 3$

$c = a \qquad b = b$

The root lies between c and b.

$a = c \qquad f(a) = f(c)$

By repeating the procedure we get :

The approximate root is :

2.7392

Example:

Solve the equation
$x \tan x = -1$ by false position method starting with 2.5 and 3.0 as the initial approximation of the root with error not more than 0.001

$f(x) = x\tan x + 1$
$f(a) = f(2.5) = -0.8675$
$f(b) = f(3) = 0.5724$

$$\therefore \quad c = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$$= 2.8012$$

$f(c) = 0.00787$

we have $f(2.5) * f(2.8012) < 0$

$\therefore$ The root lies between 2.5 and 2.8012
b a

The second approximation to the root is given by.

$$C = \frac{2.8012 . f(2.5) - 2.5 f(2.8012)}{f(2.5) - f(2.8012)}$$

$$= 2.7984$$

$f(c) = 0.000039$

$f(2.5) * f(2.7984) < 0$

∴ The root lise between 2.5 and 2.7984
b a

the third approoxmation is given by.

$$C = \frac{2.7984\, f(2.5) - 2.5\, f(2.7984)}{f(2.5) - f(2.7984)}$$

$= 2.7982$

The required root is 2.798

Home work

1- Compute the root of the equation
$x^3 - 4x - 9 = 0$ ∴ $[2, 3]$
$e = 0.01$

ans : 2.71

2. $\log_e x = \cos x$ $[1, 2]$ $\{\ln x = \cos x\}$
ans : 1.303 $e = 0.01$

3. $\sin x + \cos x = 1$ $[1, 2]$.
ans : 1.521 $e = 0.01$

4. Plot a flow chart showing the solution of equation $f(x) = 0$ where the root lies in the interval $[a, b]$ and the required error is not more than e